

Conformal Standard Model

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Abstract

In recent papers we have constructed the conformal theory of metric-torsional gravitation, and in this paper we shall include the gauge fields to study the conformal $U(1) \times SU(2)$ Standard Model; we will show that the metric-torsional degrees of freedom give rise to a potential of conformal-gauge symmetry breaking: consequences are discussed.

Introduction

The reasons for which conformal Weyl gravity is remarkable are two: at the scale of the solar system Weyl gravity solutions are able to approximate the Einstein gravity solutions but as the scales increase to the size of galaxies they are also able to fit the rotation curves [1] and for the universe in its entirety they address the cosmological constant problem [2]; the equations of Weyl gravity are fourth-order differential equations with dimensionless constants whose renormalizability is useful to address the gravitational quantization [3]. With respect to the last issue, one has to keep in mind that not only the metric but also the torsional degrees of freedom have to be considered, because according to the Wigner classification of quantum particles in terms of their mass and spin matter fields possess not only energy but also spin density: thus for a conformal metric-torsional gravitation to be constructed one needs to define beside the conformal transformations for the metric also the corresponding conformal transformations for the torsion [4]. Then one needs to define a conformally covariant metric-torsional curvature in $(1+3)$ -dimensional spacetimes upon which to build the conformal gravitational field theory [5]. This theory has further been applied to the special case of the conformal massless Dirac field theory [6].

Then it is important to recall that not only the metric-torsional degrees of freedom but also the gauge degrees of freedom have to be considered because matter fields possess not only energy-spin but also current density: for the conformal metric-torsional gravitation to include gauge fields one should define conformal transformations for the gauge fields, which nonetheless are trivial as it is widely known. Because of this fact we have that the conformally covariant gauge strength in $(1+3)$ -dimensions is unchanged with respect to the standard case in which the conformal invariance of gauge field theories was already a character of the model [7]. The inclusion of scalar fields follows [8].

However when in the purely metric curvature the torsion is accounted then the metric-torsional curvature changes its properties, and consequently the effects of its coupling to other fields, so that while the background is decoupled

from the gauge strength, and therefore such a modification has no influence on the gauge sector, the background has peculiar coupling to the scalar fields, and henceforth this modification greatly affects the scalar fields as we shall show in the present paper. This will allow us to merge the results of [9] and [10] obtaining a metric-torsion conformally invariant $U(1) \times SU(2)$ -gauge symmetric interaction for the Dirac and scalar fields to build a unique Standard Model.

1 Conformal Geometry

In this paper we shall follow all notation and conventions of [5, 6] about conformal gravity. In particular the metric $g_{\alpha\theta}$ tensor has conformal transformation

$$g_{\alpha\theta} \rightarrow \sigma^2 g_{\alpha\theta} \quad (1)$$

and the Cartan torsion $Q_{\sigma\rho\alpha}$ is a tensor with conformal transformation

$$Q_{\sigma\rho\alpha}^\sigma \rightarrow Q_{\sigma\rho\alpha}^\sigma + q\sigma^{-1}(\delta_\rho^\sigma \partial_\alpha \sigma - \delta_\alpha^\sigma \partial_\rho \sigma) \quad (2)$$

for a given parameter q which may be seen as a sort of conformal charge, and they are used to build the metric-compatible connection $\Gamma_{\alpha\nu}^\rho$ defining the covariant derivatives D_α and the Riemann-Cartan metric-torsional curvature tensor

$$G_{\xi\mu\nu}^\rho = \partial_\mu \Gamma_{\xi\nu}^\rho - \partial_\nu \Gamma_{\xi\mu}^\rho + \Gamma_{\sigma\mu}^\rho \Gamma_{\xi\nu}^\sigma - \Gamma_{\sigma\nu}^\rho \Gamma_{\xi\mu}^\sigma \quad (3)$$

as usual; then from Cartan torsion and Riemann-Cartan metric-torsional curvature we define a modified metric-torsional curvature tensor

$$M_{\alpha\theta\mu\nu} = G_{\alpha\theta\mu\nu} + (\frac{1-q}{3q})(Q_\theta Q_{\alpha\mu\nu} - Q_\alpha Q_{\theta\mu\nu}) \quad (4)$$

with the same symmetries: further we have that the irreducible decomposition is the traceless curvature tensor

$$T_{\alpha\theta\mu\nu} = M_{\alpha\theta\mu\nu} - \frac{1}{2}(M_{\alpha[\mu} g_{\nu]\theta} - M_{\theta[\mu} g_{\nu]\alpha}) + \frac{1}{12}M(g_{\alpha[\mu} g_{\nu]\theta} - g_{\theta[\mu} g_{\nu]\alpha}) \quad (5)$$

having the same symmetries for indices transposition and traceless for indices contraction but also conformally covariant. And finally it is in terms of the set of three free parameters A , B , C that it is possible to define the parametric curvature tensor

$$P_{\alpha\theta\mu\nu} = AT_{\alpha\theta\mu\nu} + BT_{\mu\nu\alpha\theta} + \frac{C}{4}(T_{\alpha\mu\theta\nu} - T_{\theta\mu\alpha\nu} + T_{\theta\nu\alpha\mu} - T_{\alpha\nu\theta\mu}) \quad (6)$$

having the same antisymmetry properties for both the first and the second pair of indices transposition and traceless for any indices contraction and conformally covariant in $(1+3)$ -dimensional spacetimes, which will be useful in the following.

Next we will follow the notation [9, 10] for the standard model. In particular the fields B_μ and \vec{A}_μ are vectors having gauge transformations given by the abelian $U(1)$ and the simplest non-abelian $SU(2)$ group and with trivial conformal transformation, from which it is possible to define the gauge-covariant derivatives D_α and the Maxwell and Yang-Mills gauge curvature tensors

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (7)$$

$$\vec{A}_{\mu\nu} = \partial_\mu \vec{A}_\nu - \partial_\nu \vec{A}_\mu + g \vec{A}_\mu \times \vec{A}_\nu \quad (8)$$

antisymmetric for indices transposition and traceless for indices contraction and conformally covariant in $(1+3)$ -dimensional spacetimes.

The fermion fields are introduced as a single right-handed spinor ψ_R and a doublet of left-handed spinors ψ_L defined in terms of their transformation law under the same $U(1) \times SU(2)$ group and with scaling $\sigma^{-\frac{3}{2}}$ as conformal transformation, with spinorial gauge-covariant derivatives defined accordingly.

The Higgs field is a doublet of complex scalar fields ϕ set by its transformation law under the same $U(1) \times SU(2)$ group and with scaling σ^{-1} as conformal transformation, with gauge-covariant derivatives defined accordingly.

2 Conformal Standard Model

For the Standard Model as we know it [9], we have that the action is a scalar and gauge symmetric under the $U(1) \times SU(2)$ group and not conformally invariant as the lagrangian has terms that do not scale by the σ^{-4} factor, but instead they scale by the σ^{-2} factor; these terms are given by the Ricci curvature G necessary for the gravitational dynamics and the Higgs quadratic potential ϕ^2 essential to bring the trivial vacuum in a non-stable configuration that is supposed to eventually move toward a stable configuration with non-trivial vacuum.

For a conformal version of the Standard Model instead [10], the action must be a scalar and gauge symmetric under the $U(1) \times SU(2)$ group and also conformally invariant with a lagrangian that scales by the σ^{-4} factor; consequently we need to have, on the one hand, terms like the square of the $T_{\alpha\theta\mu\nu}$ tensor can be used to determine the gravitational dynamics, while, on the other hand, the product between the curvature M and the quadratic potential ϕ^2 may be used to give rise to a potential of conformal-gauge symmetry breaking.

Actually the approach enjoys a particular elegance that can be appreciated by noticing the following fact: under a global conformal transformation both the Higgs dynamical term $D_\rho \phi^\dagger D^\rho \phi$ and the Higgs potential term $\phi^2 M$ scale by the correct σ^{-4} factor although under a local more general conformal transformation these two terms will be accompanied by extra pieces that would spoil the invariance, unless a proper fine-tuning is chose so to have them all cancelling exactly to yield a conformally invariant Higgs action; under this point of view then, the potential of conformal-gauge symmetry breaking is not just added for the sake of generality, but because such Higgs potential beside the Higgs dynamical term is necessary to maintain the conformal-gauge symmetry before its break-down will occur. On the other hand however, general conformal transformation in presence of metric and torsion widen the range of possibilities because beyond the term $\phi^2 M$ also $\phi^2 Q_\alpha Q^\alpha$ as well as $D_\nu \phi^2 Q^\nu$ may be used beside the dynamical term $D_\rho \phi^\dagger D^\rho \phi$ to restore the metric-torsional conformal invariance of the Higgs action: after a straightforward calculation is performed it is easy to find the most general Higgs action and therefore the total action is given in the form

$$\begin{aligned}
S_{\text{SM}} = & \int [T^{\alpha\theta\mu\nu} P_{\alpha\theta\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4} \vec{A}^{\mu\nu} \cdot \vec{A}_{\mu\nu} + \\
& + \frac{i}{2} (\bar{\psi}_R \gamma^\mu D_\mu \psi_R - D_\mu \bar{\psi}_R \gamma^\mu \psi_R) + \frac{i}{2} (\bar{\psi}_L \gamma^\mu D_\mu \psi_L - D_\mu \bar{\psi}_L \gamma^\mu \psi_L) + \\
& + D_\rho \phi^\dagger D^\rho \phi + \left(\frac{1-6k(1-q)}{3q} \right) D_\nu \phi^2 Q^\nu + \left(\frac{1-6k(1-q)(1+2q)}{9q^2} \right) \phi^2 Q_\alpha Q^\alpha + \\
& + k \phi^2 M - Y (\bar{\psi}_R \phi^\dagger \psi_L + \bar{\psi}_L \phi \psi_R) - \frac{\lambda}{8} \phi^4] \sqrt{|g|} dV
\end{aligned} \tag{9}$$

in terms of the k , λ and Y parameters and such that under the most general coordinate $U(1) \times SU(2)$ conformal transformation this action is invariant.

By varying this action with respect to the fields involved we obtain the gravitational field equations for the energy and spin

$$4[(\frac{1-q}{3q})(\frac{1}{2}Q_{\sigma\rho\beta}g^{\mu[\alpha}P^{\theta]\sigma\rho\beta} - Q_{\rho}P^{\rho[\alpha\theta]\mu}) + D_{\rho}P^{\alpha\theta\mu\rho} + Q_{\rho}P^{\alpha\theta\mu\rho} - \frac{1}{2}Q^{\mu}_{\rho\beta}P^{\alpha\theta\rho\beta}] = S^{\mu\alpha\theta} \quad (10)$$

$$2[(\frac{1-q}{3q})(Q_{\nu}(2P^{\mu\rho\alpha\nu}Q_{\rho} - g^{\mu\alpha}P^{\nu\theta\rho\sigma}Q_{\theta\rho\sigma} - P^{\mu\nu\rho\sigma}Q^{\alpha}_{\rho\sigma}) + D_{\nu}(2P^{\mu\rho\alpha\nu}Q_{\rho} - g^{\mu\alpha}P^{\nu\theta\rho\sigma}Q_{\theta\rho\sigma} + g^{\mu\nu}P^{\alpha\theta\rho\sigma}Q_{\theta\rho\sigma})) + P^{\theta\sigma\rho\alpha}T_{\theta\sigma\rho}^{\mu} - \frac{1}{4}g^{\alpha\mu}P^{\theta\sigma\rho\beta}T_{\theta\sigma\rho\beta} + P^{\mu\sigma\alpha\rho}M_{\sigma\rho}] + \frac{1}{2}[(B^{\alpha\rho}B^{\mu}_{\rho} - \frac{1}{4}B^2g^{\alpha\mu}) + (\vec{A}^{\alpha\rho} \cdot \vec{A}^{\mu}_{\rho} - \frac{1}{4}A^2g^{\alpha\mu})] = \frac{1}{2}T^{\alpha\mu} \quad (11)$$

and the gauge field equations for both currents

$$D_{\rho}B^{\rho\mu} + Q_{\rho}B^{\rho\mu} + \frac{1}{2}Q^{\mu\beta\rho}B_{\beta\rho} = J^{\mu} \quad (12)$$

$$D_{\rho}\vec{A}^{\rho\mu} + Q_{\rho}\vec{A}^{\rho\mu} + \frac{1}{2}Q^{\mu\beta\rho}\vec{A}_{\beta\rho} = \vec{J}^{\mu} \quad (13)$$

where the energy and spin are given by

$$S^{\mu\alpha\theta} = \frac{i}{4}\bar{\psi}_R\{\gamma^{\mu}, \sigma^{\alpha\theta}\}\psi_R + \frac{i}{4}\bar{\psi}_L\{\gamma^{\mu}, \sigma^{\alpha\theta}\}\psi_L + \left(\frac{6k-1}{6q}\right)(D^{\theta}\phi^2g^{\alpha\mu} - D^{\alpha}\phi^2g^{\theta\mu}) + \left(\frac{1-6k-3kq^2}{9q^2}\right)\phi^2(Q^{\alpha}g^{\theta\mu} - Q^{\theta}g^{\alpha\mu}) - k\phi^2Q^{\mu\alpha\theta} \quad (14)$$

$$T^{\alpha\mu} = \frac{i}{2}(\bar{\psi}_R\gamma^{\alpha}D^{\mu}\psi_R - D^{\mu}\bar{\psi}_R\gamma^{\alpha}\psi_R) + \frac{i}{2}(\bar{\psi}_L\gamma^{\alpha}D^{\mu}\psi_L - D^{\mu}\bar{\psi}_L\gamma^{\alpha}\psi_L) + (D^{\alpha}\phi^{\dagger}D^{\mu}\phi + D^{\mu}\phi^{\dagger}D^{\alpha}\phi - g^{\alpha\mu}D_{\rho}\phi^{\dagger}D^{\rho}\phi) - \left(\frac{1-6k(1-q)}{3q}\right)(D^{\mu}D^{\alpha}\phi^2 - D^2\phi^2g^{\alpha\mu} - Q^{\alpha}D^{\mu}\phi^2) - 2\left(\frac{1-6k(1-q^2)}{9q^2}\right)(D^{\mu}\phi^2Q^{\alpha} - g^{\alpha\mu}D_{\nu}\phi^2Q^{\nu} + \phi^2D^{\mu}Q^{\alpha} - g^{\alpha\mu}\phi^2D_{\rho}Q^{\rho}) + \left(\frac{1-6k(1-q)}{9q^2}\right)g^{\alpha\mu}\phi^2Q^{\nu}Q_{\nu} - 2k\left(\frac{1-q}{3q}\right)\phi^2(Q^{\alpha}Q^{\mu} + Q^{\alpha\mu\theta}Q_{\theta}) + 2k\phi^2(M^{\alpha\mu} - \frac{1}{2}g^{\alpha\mu}M) + \frac{\lambda}{8}g^{\alpha\mu}\phi^4 \quad (15)$$

and the two currents are given by

$$J^{\mu} = -g'\bar{\psi}_R\gamma^{\mu}\psi_R - \frac{g'}{2}\bar{\psi}_L\gamma^{\mu}\psi_L - \frac{ig'}{2}(D^{\mu}\phi^{\dagger}\phi - \phi^{\dagger}D^{\mu}\phi) \quad (16)$$

$$\vec{J}^{\mu} = -\frac{g}{2}\bar{\psi}_L\gamma^{\mu}\vec{\sigma}\psi_L + \frac{ig}{2}(D^{\mu}\phi^{\dagger}\vec{\sigma}\phi - \phi^{\dagger}\vec{\sigma}D^{\mu}\phi) \quad (17)$$

complemented by the Dirac field equations

$$i\gamma^{\mu}D_{\mu}\psi_R + \frac{i}{2}Q_{\mu}\gamma^{\mu}\psi_R - Y\phi^{\dagger}\psi_L = 0 \quad (18)$$

$$i\gamma^{\mu}D_{\mu}\psi_L + \frac{i}{2}Q_{\mu}\gamma^{\mu}\psi_L - Y\phi\psi_R = 0 \quad (19)$$

and scalar field equations

$$D^2\phi + Q^{\rho}D_{\rho}\phi + \left(\frac{1-6k(1-q)}{3q}\right)D_{\nu}Q^{\nu}\phi + \left(\frac{-1+3q+6k-12qk+6kq^2}{9q^2}\right)Q^{\nu}Q_{\nu}\phi - kM\phi + \frac{\lambda}{4}\phi^2\phi + Y\bar{\psi}_R\psi_L = 0 \quad (20)$$

constituting the complete set of field equations of the conformal Standard Model.

Finally it is possible to see that in this set of field equations when the Dirac and scalar field equations are considered for the energy and spin and the two currents then the conserved quantities satisfy the following conservation laws arising from the invariance under gauge phase transformations

$$D_\mu J^\mu + Q_\mu J^\mu = 0 \quad (21)$$

$$D_\mu \vec{J}^\mu + Q_\mu \vec{J}^\mu = 0 \quad (22)$$

the spacetime frame transformations

$$D_\mu T^{\mu\rho} + Q_\mu T^{\mu\rho} - T_{\mu\sigma} Q^{\sigma\mu\rho} + S_{\theta\mu\sigma} G^{\sigma\mu\theta\rho} + J_\mu B^{\mu\rho} + \vec{J}_\mu \cdot \vec{A}^{\mu\rho} = 0 \quad (23)$$

$$D_\rho S^{\rho\mu\nu} + Q_\rho S^{\rho\mu\nu} + \frac{1}{2} T^{[\mu\nu]} = 0 \quad (24)$$

and spacetime conformal scaling

$$(1 - q)(D_\mu S_\nu{}^{\nu\mu} + Q_\mu S_\nu{}^{\nu\mu}) + \frac{1}{2} T_\mu{}^\mu = 0 \quad (25)$$

and for which the Jacobi-Bianchi identities are verified identically as expected.

2.1 Higgs Potential for Symmetry Breaking

Conformal Standard Models as those built so far have an advantage with respect to the ordinary Standard Model [9], which consists in the fact that because of the presence of conformal gravitational degrees of freedom there is a more complicated form for the Higgs potential that induces the symmetry breaking phenomenon; in fact after a direct calculation we have that once it is given the vacuum expectation value $v^2 = \phi^2$ for the Higgs field then the condition for the stable stationary point for which the symmetry is broken is given by

$$\frac{\lambda}{4} v^2 = - \left(\frac{1-6k(1-q)}{3q} \right) D_\nu Q^\nu - \left(\frac{-1+3q+6k-12qk+6kq^2}{9q^2} \right) Q^\nu Q_\nu + kM \quad (26)$$

which we are now going to discuss in some detail.

Notice first of all that in the purely metric case condition (26) reduces to the simplest $4kR = \lambda v^2$ which is indeed satisfied in a de Sitter spacetime of negative spatial curvature as reported for instance in [10]; however as observations point toward the fact that the spatial curvature is approximately vanishing then the de Sitter spacetime is approximately flat and the above condition yields a vacuum expectation value approximately zero, and so even in the case in which such a vacuum expectation value were different from zero as to produce symmetry breaking it would not be large enough to reach the Linde-Weinberg lower bound that is necessary to produce a stable breaking of the underlying symmetry.

Thus it is necessary to assume that metric is accompanied by torsion in condition (26) so that even in cases of cosmological relevance given when we have the constraint $R = 0$ this condition would still be not degenerate; because in these cosmological applications torsion can be decomposed in terms of trace and completely antisymmetric parts alone then we have

$$\frac{\lambda}{4} v^2 = \left(\frac{6k-1}{3q} \right) D_\nu Q^\nu + \left(\frac{6k-1}{3q} \right) \left(\frac{3q-1}{3q} \right) Q^\nu Q_\nu + \frac{3k}{2} V^\alpha V_\alpha \quad (27)$$

in which the Q_ν and V^ρ are respectively the vector trace and pseudo-vector dual of the completely antisymmetric parts of the torsion tensor, and this condition can actually be simplified a little further as we are going to show now.

In fact by noticing that if the vector trace were to vanish or if the fine-tuning given by $k = \frac{1}{6}$ were to be chosen then the condition (27) would reduce to the simpler $V^\alpha V_\alpha = \lambda v^2$ telling that it is in terms of V^ρ pseudo-vector dual of the completely antisymmetric part of the torsion tensor that the vacuum expectation value is given; because $V^\rho V_\rho = V^2$ may be either positive or negative then the constant λ may be either positive or negative as well, and still we would be able to get a real value v for the ground state of the Higgs field, with the consequent symmetry breaking occurring as in the Standard Model.

2.2 Generation of Mass and Cosmological Constant

As it is widely known, after the vacuum expectation value is gotten by the Higgs field a symmetry breaking occurs because the new ground state of the Higgs field is no longer invariant, and the new Higgs field is seen as a fluctuation over the special ground state of the type, producing two mechanisms: on the one hand, there is generation of the masses of the particles that couple to the Higgs, taking place in two ways: both as a transfer of degrees of freedom from the Higgs to the massless bosons which then become massive bosons, and as the result of the presence of the potential of interaction between Higgs and fermions and of self-interaction of the Higgs with itself; on the other hand, there is the appearance of the cosmological constant, again as the result of the presence of the same potential of self-interaction of the Higgs with itself. In the following, we shall not take into account the mechanism of the generation of the masses of the bosons, thoroughly discussed in the literature; we will instead focus on the generation of the masses of the fermion and Higgs field and of the cosmological constant, whose origin is due to the presence of the Yukawa and Higgs potentials.

After a straightforward calculation, it is easy to see that the values of the mass of the fermion and the Higgs and also the cosmological constant

$$m_{\text{fermion}} = Yv \quad m_{\text{Higgs}}^2 = \frac{\lambda v^2}{2} \quad \Lambda = \frac{\lambda v^4}{16} \quad (28)$$

are given by the Yukawa coupling Y and the Higgs parameter λ in terms of the vacuum expectation value v of the Higgs field itself. Notice that as the Yukawa coupling is unknown the knowledge of the fermion mass does not give any clue about v whose value of about 350 GeV is determined when the low-energy limit of the fermion scattering is compared to the effective Fermi scattering, and this value is used to evaluate from the Higgs mass the cosmological constant.

In fact by combining the two definitions above we have that

$$\Lambda = \left(\frac{m_{\text{Higgs}} v}{2\sqrt{2}} \right)^2 \quad (29)$$

which with the vacuum expectation value of about 350 GeV and the Higgs mass at least of the order of magnitude of 10^2 GeV gives a cosmological constant at least of order of magnitude of 10^8 GeV⁴ which is far from the upper limit of the order of magnitude of 10^{-46} GeV⁴ as astrophysical experiments tell.

This situation, in which the ground state of the Higgs field gives reasonable masses only at the price of having a largely wrong cosmological constant, gives

rise to what is usually known as the cosmological constant problem, a problem that the present model may solve by exploiting the fact that the Higgs potential here is much more complex than the quartic one of the Standard Model.

To see this, we take into account the spin density in its completely antisymmetric part and employing it to calculate the ground state we find that

$$\begin{aligned} \lambda v^6 = & -\frac{9}{4} (\bar{e} \gamma^\rho e \bar{e} \gamma_\rho e + 2 \bar{\nu} \gamma^\rho e \bar{e} \gamma_\rho \nu + 4 \bar{\nu} e \bar{e} \nu) + \\ & + 3 (\bar{e} \gamma^\rho \gamma e - \bar{\nu} \gamma^\rho \nu) S^{\mu\alpha\theta} \varepsilon_{\mu\alpha\theta\rho} + S^{\mu\alpha\theta} S_{\nu\zeta\sigma} \varepsilon_{\mu\alpha\theta\rho} \varepsilon^{\nu\zeta\sigma\rho} \end{aligned} \quad (30)$$

in terms of the electron and neutrino fields; when the field equations for the spin density are accounted, the spin density is given in terms of torsion-curvature conformal coupling: in this condition we read that the vacuum of the Higgs field is related to both the fermion density and some conformal geometrical quantities, the former having much more influence in particle physics than in cosmology while the latter being supposed to have the same effects at any scale, and therefore implying that the ground state of the Higgs field should be much larger in particle physics than in cosmological applications.

So to summarize, we have that the mechanism that generates the masses also gives a cosmological constant whose value is too large compared to observations, and this problem may be solved by a model in which the vacuum expectation value tends to vanish moving from particle physics toward cosmology: in the present torsional-metric conformal Standard Model there is a possible solution because the vacuum is related to a dynamical field that is itself related to the fermion density and therefore relevant in particle physics and nearly negligible in cosmology; in the purely metric conformal Standard Model there is a solution as well if we consider a de Sitter spacetime with negative spatial curvature of a given radius, but for this solution to work the spatial curvature radius has to be adapted to one value for particle physics and another value for cosmology, which may be possible given that the spatial curvature radius is a parameter, but it is hard to imagine how the spatial curvature radius should depend on the physical situation we want to study; and in the Standard Model the ground state is a constant, and therefore fixed once and for all at any scales, whether we are in particle physics or cosmology, and thus having no way to quench the discrepancies between particle physics and cosmology. In other words, we may say that the cosmological constant problem may be solved by a ground state that depends on the energy scales of the situation: in the present metric-torsionless conformal Standard Model v is dynamical and thus naturally suited for this purpose, in the purely metric conformal Standard Model v is a parameter addressing the issue only at the cost of tuning its value in an unnatural way, and in the Standard Model v is a constant thus leaving no room for any solution of the problem whatsoever.

Conclusion

In the present paper, we have considered the fully endowed metric-torsional conformal Standard Model writing the most generally invariant action and deriving the field equations: we have isolated the Higgs sector determining the ground state for the stable stationary potential; we have discussed the vacuum expectation value in some cases such as de Sitter spacetimes with trace and

completely antisymmetric parts of torsion, calculating the generated masses and the cosmological constant, and discussing how the cosmological constant problem eventually arises. The results we have found indicate that the cosmological problem may be solved by a model in which the vacuum expectation value tends to vanish as we move from particle physics toward cosmology, and we have discussed how this is impossible in the Standard Model but possible in the conformal Standard Model; we have stressed that in the purely metric conformal Standard Model such a solution is possible only if we insist to have a scale-dependent spatial curvature of the spacetime that admittedly does not fit into a uniquely defined spacetime whereas in the torsional-metric conformal Standard Model the solution is possible in general because torsion is a dynamical field related to quantities that are present in particle physics while vanishing in cosmology, and the cosmological constant problem is solved naturally.

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